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Abstract: Numerical simulations of unsteady flows past a flexible structure require the simultaneous solution of structural dynamics and of fluid dynamics on deformable dynamic meshes. We present here an evaluation method of staggered partitioned procedures for time-integrating these focus coupled problems. This method is based on an estimation of the energy that is numerically created at the fluid/structure interface because of staggering. Simplifying assumptions make this estimation easy for a wide family of mixed explicit/implicit or implicit/implicit, synchronous or asynchronous procedures. Insights gained from this evaluation method are confirmed with the analysis of numerical results for the flutter of a flat panel in supersonic airstreams in two and three dimensions and for the transient aeroelastic response of an AGARD 445.6 wing in the transonic regime.

Key-words: Fluid-structure interactions, coupling algorithm, staggered partitioned procedure, energy conservation, evaluation and design criterion

(Résumé : tsvp)

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Construction et évaluation de procédures décalées pour la simulation d'interactions fluide-structure

Résumé : La simulation numérique d'écoulements intationnaires autour de structures souples nécessite la résolution simultanée de problèmes de dynamique des structures et de dynamique des fluides en domaine déformable. Nous présentons ici une méthode d'évaluation d'algorithmes modulaires décalés utilisés pour l'intégration en temps de ce type de problème. Cette méthode repose sur une estimation de l'energie créée numériquement à l'interface fluide-structure à cause du décalage. Des hypothèses simplificatrices permettent d'évaluer simplement cette energie pour une grande famille d'algorithmes. Les prédictions obtenues par cette méthode ont été confirmées par l'analyse des résultats de simulations aéroélastiques du flottement d'un panneau soumis à un écoulement supersonique en deux et trois dimensions, et de la réponse instationnaire d'une aile AGARD 445.6 en régime transsonique.

Mots-clé : Interactions fluide-struture, algorithme de couplage, schémas décalés, conservation de l'énergie, critère d'évaluation/construction

1 Introduction

The numerical simulation of fluid/structure interaction phenomena arises in many aerospace engineering applications such as airfoil oscillations, flutter predictions, fighter tail buffeting, and a large class of other aeroelastic instability problems. Although closed-form solutions are available for aeroelastic computations when flows are not in the transonic regime, computational methods for complex nonlinear flows have been under development for almost twenty years [7]. These methods should be as efficient as possible (maximal time step for transient analysis). However, they should also predict accurately systems instabilities (the numerical diffusion - and then the time step - should be small), such that aircraft designer can rely on numerical simulations. Partitioned procedures for the transient analysis of fluid/structure interaction have become very popular and, more precisely, staggered procedures, for which Fluid Dynamics and Structural Dynamics are time integrated separately and successively. This kind of strategy allows modularity and computational efficiency through possible interfield parallelism [5].

In the past years, physical accuracy of transient numerical simulations was achieved by reducing the time step and as a side-effect the computational efficiency. The increase of machine performances was a partial solution to this problem. At the same time, some progress has been done towards the construction of accurate and efficient methods [12]. The role of energy and momentum exchanges through the fluid/structure interface has been emphasized. For example, the inconditional stability of some particular linear coupled system as well as true staggered procedure for these systems has been established, when some energy conservation property was assumed [13]. Furthermore, these ideas were translated to nonlinear configurations, like the inviscid flow past a flexible structure, to improve the accuracy and stability properties of some partitioned procedures [11].

The aim of this paper is to construct a criterion for partitioned procedures giving an evaluation of the energy that is artificially created at the interface because of staggering. In order to make this evaluation easy, this energy is estimated under simplifying assumptions that might be dropped in further developements. An evaluation parameter is proposed for a wide family of staggered partitioned procedures (Section 2). These *a priori* estimations are then compared with numerical results of aeroelastic simulations in two- and three-dimensional configurations (Section 3).

2 Principles and description of the evaluation

In this section, we present the evaluation method for staggered partitioned procedures. These methods have been introduced to perform in a simple way numerical simulations of coupled problems. They computationally couple numerical methods for the interacting subsystems. However, the stability and accuracy of simple coupling algorithms are very different from those of numerical methods used in each decoupled field. In general, a much stronger condition on the time step is required. This explains why such computations can be very expensive.

It has been shown that the loss in accuracy and stability is due to staggering, because the forces and energies exchanged at the fluid/structure interface are not balanced [11]. Furthermore, a gain in accuracy and stability can be obtained if the differences between exchanged energies is controlled [11]. We propose here to construct an evaluation of the energy that is actually created at the interface because of staggering. We first compare the accumulated energies exchanged between the fluid and the structure for synchronous and asynchronous staggered procedures. Thereafter, we discuss the relative merits of each family of procedures as predicted by the energetic evaluation.

We first fix some notation. In the following, U denotes structural displacements and a dotted symbol stands for the time-derivative of a quantity. We consider a generic point at the fluid/structure interface, and the corresponding boundary element. We omit the length or surface of this small element and the symbol P (for pressure) stands for a force as well (in the case of a viscous flow, the fluid force is not limited to the pressure force). Finally, superscripts are always related to the time, and the time step Δt is the structural time step.

2.1 Evaluation for a class of synchronous staggered schemes

We first consider the conventional serial staggered procedure (CSS) as introduced by Farhat *et al* [4]. This procedure is synchronous, i.e. the fluid and the structure are computed at the same times. In the following, we shall also consider the leap-frog type improved serial staggered procedure (ISS) of [4], which is clearly asynchronous. To make the description a little shorter, we now give the details of a generic staggered algorithm. For the staggered integration from t^n to $t^{n+1} = t^n + \Delta t$, the CSS prodecure reads

1. make a prediction X^{n+1} for the structural displacement at time t^{n+1}. In many studies, this prediction is completely outdated and reads $X^{n+1} = U^n$. We consider the more general prediction in function of parameters α_0 and α_1,

$$X^{n+1} = U^n + \alpha_0 \Delta t \dot{U}^n + \alpha_1 \Delta t \left(\dot{U}^n - \dot{U}^{n-1} \right) \tag{1}$$

2. compute a new fluid grid at time t^{n+1} matching this predicted displacement, and advance the fluid of Δt, possibly in a subcycled way (with as many fluid time steps as necessary). For subcycling, we have limited our investigations to algorithms for which the mesh speed is constant during a coupled time step. It is then given by $w^{n+1/2} = (X^{n+1} - X^n)/\Delta t$. This is the case for all procedures considered in [4], in which this choice was actually advocated.

3. compute a distribution of transferred fluid pressure forces P_S^{n+1} applied to the structure (with the time-averaging or time scheme of your choice). This force can be seen as the pressure force exerted by the fluid on the structure at time t^{n+1}.

4. advance the structure of Δt with the previously computed force.

The prediction (1) is first-order accurate if $\alpha_0 = 1$ and second-order accurate under the additional condition that $\alpha_1 = 1/2$. The fluid time-integration of step 2. can be done with explicit or implicit, first- or second-order accurate time schemes. In all cases, we can give an estimate for the transferred energy through an element of the fluid/structure interface. This estimate has the general form

$$\Delta E_F = -\left({}^T X^{n+1} - {}^T X^n\right) P_F^n, \tag{2}$$

where X denote successive displacements of the fluid grid boundary (coinciding with the prediction of the structural displacements) and P_F^n is the fluid pressure involved in the boundary flux for the current time-step. This formula is exact in one-dimension. P_F^n can take one of the following values:

$$P_F^n = P^n \tag{3a}$$

$$P_F^n = P^{n+1} \tag{3b}$$

$$P_F^n \sim \frac{P^n + P^{n+1}}{2} \tag{3c}$$

$$P_F^n \sim \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} P(t)dt \tag{3d}$$

For example, for a first-order forward-Euler scheme, the energy flux through the fluid is computed using fluid values at time t^n. We then have $\Delta E_F = -\Delta t\ {}^T w^{n+1/2} P_n = \left({}^T X^{n+1} - {}^T X^n\right) P_n$. Then P_F^n is given by (3a). For an implicit backward-Euler scheme, P_F^n is given by (3b). This equation is only exact for true un-linearized implicit versions of the backward-Euler scheme. For second-order time schemes, a good approximation of P_F^n is given by (3c). Finally, we have considered the possibility of fluid subcycling. We assume the fluid grid is constant during all fluid subcycles of a coupled time-step. When the fluid time-step is sufficiently small compared with the structural time-step Δt, a second-order estimate of P_F^n is given by (3d).

Finally, in step 4., we assume the structure is linear and discretized using a finite element model yielding mass, damping and stiffness matrices M_s, D_s, and K_s (all three are symmetric positive, M_s and K_s being definite positive). It is time-advanced using the implicit trapezoidal rule, written in function of structural displacements U, velocities $V = \dot{U}$ and accelerations $A = \ddot{U}$, as

$$\left\{\begin{array}{l} U^{n+1} = U^n + \Delta t \frac{V^n + V^{n+1}}{2} \\ V^{n+1} = V^n + \Delta t \frac{A^n + A^{n+1}}{2} \\ M_s\ A^{n+1} + D_s\ V^{n+1} + K_s\ U^{n+1} = P_S^{n+1} \end{array}\right. \tag{4}$$

One fundamental property of the preceding trapezoidal rule concerns the energy received by the structure during one time step. We consider the structural energy

$$E_S = \frac{1}{2}{}^T V M_s V + \frac{1}{2}{}^T U K_s U, \tag{5}$$

then the structural energy variation during a time step is

$$E_S^{n+1} - E_S^n = \left(U^{n+1} - U^n\right) \frac{P_S^n + P_S^{n+1}}{2} - \Delta t.{}^T V^{n+1/2} D_s V^{n+1/2},$$

where $V^{n+1/2} = (V^n + V^{n+1})/2$. We can make a distinction between the energy lost by internal damping (negative term $-\Delta t.{}^T V^{n+1/2} D_s V^{n+1/2}$) and the energy transferred by the fluid

$$\Delta E_S = \left({}^T U^{n+1} - {}^T U^n\right) \frac{P_S^n + P_S^{n+1}}{2}. \tag{6}$$

Clearly, the exchanged energies (6) and (2) with P_F^n given by (3) cannot compensate exactly because the prediction (1) is not exact. Thus, energy is artificially created or dissipated at the fluide/structure interface. This could pollute numerical results of flutter simulations for example, and yield a poor accuracy in the determination of flutter limits.

In the sequel, we intend to give an evaluation of the artificial energy creation $\Delta E_F + \Delta E_S$ in the particular case where the structure is oscillating with a constant amplitude and pulsation ω. We also assume that the fluid pressure on the structure oscillates with the same pulsation. For example, the two following coupled harmonic evolutions are possible :

$$\left\{ \begin{array}{l} U(t) = \boldsymbol{U} \cos(\omega t) \\ P(t) = \boldsymbol{P} \cos(\omega t + \varphi) \end{array} \right. \qquad \left\{ \begin{array}{l} U(t) = \boldsymbol{U} \cos(\omega t) \\ P(t) = M\ddot{U}(t) + D\dot{U}(t) + KU(t) \end{array} \right. \tag{7}$$

In (7), the first set of equations represents a simple phase shift between the fluid and the structure. In the second set of equations, aerodynamic loads on the structure have the effect of added mass, damping and stiffness matrices (respectively denoted by M, D and K). Both sets are equivalent if $\boldsymbol{P} \cos\varphi \equiv \left(K - \omega^2 M\right) \boldsymbol{U}$ and $\boldsymbol{P} \sin\varphi \equiv \omega D \boldsymbol{U}$. Let us define two parameters k and d by

$$k = {}^T\boldsymbol{U} \left(K - \omega^2 M\right) \boldsymbol{U} \equiv {}^T\boldsymbol{U} \boldsymbol{P} \cos\varphi, \quad d = {}^T\boldsymbol{U} \left(D\omega\right) \boldsymbol{U} \equiv {}^T\boldsymbol{U} \boldsymbol{P} \sin\varphi. \tag{8}$$

We assume systems are advanced in time using a constant time step Δt and successives values for U^n and P^n are then approximations of $U(t^n)$ and $P(t^n)$ with $T^n = n\Delta t$. Let us introduce the discretization parameter $h = \omega \Delta t$. Because of staggering, exchanged energies through the interface are not exactly opposite anymore. The error in energy exchanges depends on the global coupling algorithm. For example, let us assume we use the prediction (1) with $\alpha_0 = \alpha_1 = 0$ and an explicit forward-Euler scheme for the fluid with no subcycling, yielding a fluid energy variation given by (2) and (3a). Then for each time step,

$$\begin{aligned} \Delta E_F &= -\left({}^T U^n - {}^T U^{n-1}\right) P^n \\ &= \left[k \cos(\omega t^n) - d \sin(\omega t^n)\right] \left(\cos(\omega t^{n-1}) - \cos(\omega t^n)\right) \end{aligned}$$

The preceding formula is summed up over N coupled periods $T_\omega = 2\pi/\omega$ of oscillation. Using the lemmas,

$$\left\{ \begin{array}{lcl} \sum_{NT_\omega} \cos(\omega t^n + c)\cos(\omega t^n) & \sim & \frac{N\pi}{h}\cos(c) \\ \sum_{NT_\omega} \sin(\omega t^n + c)\sin(\omega t^n) & \sim & \frac{N\pi}{h}\cos(c) \\ \sum_{NT_\omega} \sin(\omega t^n + c)\cos(\omega t^n) & \sim & \frac{N\pi}{h}\sin(c) \end{array} \right. , \tag{9}$$

the energy transferred to the fluid sums up to

$$\sum_{NT_\omega} \Delta E_F \sim N\pi\ \delta E_F \quad \text{with} \quad \delta E_F = \frac{1}{h}\left[k\left(\cos(h)-1\right) - d\sin(h)\right].$$

This term can be developped assuming $h \equiv \omega\Delta t \ll 1$ and takes the form

$$\delta E_F = -k\left(\frac{h}{2} - \frac{h^3}{24}\right) - d\left(1 - \frac{h^2}{6}\right) + O(h^4).$$

Similar elementary calculations can be done with the energy variation ΔE_S of (6) for any kind of scheme. For example, if we actually advance the structure before the fluid, i.e. with $P_S^{n+1} = P^n$, then we find that the energy transferred by the fluid to the structure sums up to

$$\sum_{NT_\omega} \Delta E_S \sim N\pi\ \delta E_S \quad \text{with} \quad \delta E_S = \frac{1}{h} d\sin(h) = d\left(1 - \frac{h^2}{6}\right) + O(h^4).$$

Thus, for this particular coupling algorithm, the total energy artificially created through the fluide/structure interface sums up to

$$\sum_{NT_\omega} (\Delta E_F + \Delta E_S) \sim N\pi\ \delta E \quad \text{with} \quad \delta E \sim -k\frac{h}{2}.$$

The parameter $\delta E \equiv \delta E_F + \delta E_S$ gives an evaluation of the accuracy of the coupling algorithm. Depending on the sign of k (and also the order of magnitude of k, which depends on the form of aerodynamic loads on the structure), this parameter can help predicting the behaviour of the scheme (artificial positive or negative damping) for small time steps ($h \equiv \omega\Delta t \ll 1$).

Let us emphasize the fact that evaluation parameters δE_F and δE_S can be computed separately for different fluid and structural parts of the coupling algorithms. In the following, we give the computed parameters for different schemes and discuss the merits of possible coupling algorithms in light of the parameter δE. In the sequel, we shall say that a coupling algorithm is n^{th}-order energy-accurate if $\delta E \sim C\ h^n$ when $h \to 0$ (and C is a constant).

2.2 Evaluation of δE_F for different fluid time schemes

We now give the values of δE_F for the general prediction (1) and different fluid time schemes, including those considered above and yielding fluid energy variation (2) with P_F^n given by

(3). These values are given by

$$\begin{aligned}
\delta E_F\left[(1)(3a)\right] &= k\left[(\alpha_0-\frac{1}{2})h+(\frac{1}{24}-\frac{\alpha_0}{6}+\alpha_1)h^3\right] \\
&\quad +d\left[-1+(\frac{1}{6}-\frac{\alpha_0}{2}+\alpha_1)h^2\right]+O(h^4) \qquad (10a)\\
\delta E_F\left[(1)(3b)\right] &= k\left[(\alpha_0-\frac{3}{2})h+(\frac{5}{8}-\frac{7\alpha_0}{6}+2\alpha_1)h^3\right] \\
&\quad +d\left[-1+(\frac{7}{6}-\frac{3\alpha_0}{2}+\alpha_1)h^2\right]+O(h^4) \qquad (10b)\\
\delta E_F\left[(1)(3c)\right] &= k\left[(\alpha_0-1)h+(\frac{1}{3}-\frac{2\alpha_0}{3}+\frac{3\alpha_1}{2})h^3\right] \\
&\quad +d\left[-1+(\frac{2}{3}-\alpha_0+\alpha_1)h^2\right]+O(h^4) \qquad (10c)\\
\delta E_F\left[(1)(3d)\right] &= k\left[(\alpha_0-1)h+(\frac{1}{4}-\frac{7\alpha_0}{12}+\frac{3\alpha_1}{2})h^3\right] \\
&\quad +d\left[-1+(\frac{7}{12}-\alpha_0+\alpha_1)h^2\right]+O(h^4) \qquad (10d)
\end{aligned}$$

2.3 Evaluation of δE_S for different transferred forces P_S^{n+1}

We now give evaluations of the coefficient δE_S deriving from (6) and corresponding to different choices for the input forces P_S^{n+1} in the trapezoidal rule (4). We consider the four following choices:

$$P_S^{n+1}=P^n \qquad (11a)$$

$$P_S^{n+1}=P^{n+1} \qquad (11b)$$

$$P_S^{n+1}=\frac{P^n+P^{n+1}}{2} \qquad (11c)$$

$$P_S^{n+1}=\frac{1}{\Delta t}\int_{t^n}^{t^{n+1}}P(t)dt \qquad (11d)$$

For each choice of the form $P^\star$, we can compute a different input pressure force by determining the transferred force P_S^{n+1} verifying $\frac{P_S^n+P_S^{n+1}}{2}=P^\star$. This kind of algorithm gives an exact control of the momentum transferred by the fluid to the structure and partial control

for the energy through (6). These additional choices correspond to

$$P_S^{n+1} = 2\ P^n - P_S^n \tag{12a}$$

$$P_S^{n+1} = 2\ P^{n+1} - P_S^n \tag{12b}$$

$$P_S^{n+1} = P^{n+1} \tag{12c}$$

$$P_S^{n+1} = \frac{2}{\Delta t}\int_{t^n}^{t^{n+1}} P(t)dt - P_S^n \tag{12d}$$

The reader can notice that choices (11b) and (12c) are strictly equivalent. We give developments of the parameter δE_S in function of h for these eight (actually seven) force inputs. Tedious calculations yield

$$\delta E_S\left[(11a)\right] = k\left[-\frac{h}{2}+\frac{h^3}{3}\right] + d\left[1-\frac{2h^2}{3}\right] + O(h^4) \tag{13a}$$

$$\delta E_S\left[(11b/12c)\right] = d\left[1-\frac{h^2}{6}\right] + O(h^4) \tag{13b}$$

$$\delta E_S\left[(11c)\right] = k\left[-\frac{h}{4}+\frac{h^3}{6}\right] + d\left[1-\frac{5h^2}{12}\right] + O(h^4) \tag{13c}$$

$$\delta E_S\left[(11d)\right] = k\left[-\frac{h}{2}+\frac{h^3}{8}\right] + d\left[1-\frac{h^2}{3}\right] + O(h^4) \tag{13d}$$

$$\delta E_S\left[(12a)\right] = k\left[-\frac{h}{2}+\frac{h^3}{24}\right] + d\left[1-\frac{h^2}{6}\right] + O(h^4) \tag{13e}$$

$$\delta E_S\left[(12b)\right] = k\left[\frac{h}{2}-\frac{h^3}{24}\right] + d\left[1-\frac{h^2}{6}\right] + O(h^4) \tag{13f}$$

$$\delta E_S\left[(12d)\right] = d\left[1-\frac{h^2}{12}\right] + O(h^4) \tag{13g}$$

2.4 Discussion on δE for synchronous partitioned procedures

We first consider algorithms with $\alpha_0 = 0$ and $\alpha_1 = 0$ in the structural prediction (1). In that case, the prediction is only consistant. This kind of algorithm is first-order energy-accurate for all pair of schemes, excepted for the first-order explicit not-subcycled fluid scheme giving (3a) coupled with (12c). In that case, the error is almost zero since $\Delta E_F(t^{n-1} \rightarrow t^n) = -\Delta E_S(t^n \rightarrow t^{n+1})$. But this scheme is obviously not general.

We now consider first order predictions (1) with $\alpha_0 = 1$ and $\alpha_1 = 0$. Several algorithms in this class are second-order energy-accurate. Generally, the smallest error is obtained when exact exchange of momentum through the fluid/structure interface is achieved , i.e. when the force term in (12) corresponding to the fluid scheme is chosen. We get

$$\delta E\left[(1)_{\alpha_0=1}^{\alpha_1=1}\ -\ (3i)\ -\ (12i)\right] = -d\frac{h^2}{2} + O(h^3), \quad i = 1,\ldots,4. \tag{14}$$

However, the optimal scheme seems to be given by (3c) and 12d) for which the energy error is slightly smaller ($\delta E = -d\ 5h^2/12 + O(h^3)$).

Finally, the second order prediction (1) with $\alpha_0 = 1$ and $\alpha_1 = 1/2$ yields a third-order energy-accurate algorithm if and only if exact exchange of momentum through the fluid/structure interface is achieved. We get

$$\delta E \left[(1)_{\alpha_0=1}^{\alpha_1=1/2} - (3i) - (12i) \right] = k \frac{5h^3}{12} + O(h^4), \quad i = 1, \ldots, 4. \tag{15}$$

For the second-order prediction, the energy artificially created because of staggering is optimally reduced when the forces exchanged between the fluid and the structure at their interface are exactly opposed, and conservation of momentum is achieved. This happens when P_F^n and P_S^{n+1} respectively involved in (2) and (6) verify

$$P_F^n = \frac{P_S^n + P_S^{n+1}}{2}. \tag{16}$$

2.5 Evaluation for an asynchronous partitioned procedure

Farhat et al. [4] have advocated the use of the ISS asynchronous procedure. It is built as a leap-frog scheme (fluid values are computed at times $t^{n+1/2}$ and structural states at times t^n). The algorithm reads

1. compute the prediction $X^{n+1/2} = U^n + \frac{\Delta t}{2}\dot{U}^n$ (structural displacement at time $t^{n+1/2}$)
2. compute a new fluid grid at time $t^{n+1/2}$ matching this predicted displacement, and advance the fluid of Δt, possibly in a subcycled way.
3. compute a transferred fluid pressure distribution P_S^{n+1} applied to the structure.
4. advance the structure of Δt with the trapezoidal rule (4).

This procedure satisfies the Geometric Conservation Law (GCL) [14] without violating the interface continuity condition on velocities. This property is due to the trapezoidal rule, since $\dot{w}^n \equiv (X^{n+1/2} - X^{n-1/2})/\Delta t = (U^n - U^{n-1})/\Delta t + (\dot{U}^n - \dot{U}^{n-1})/2 \equiv \dot{U}^n$. The energy transferred to the fluid can again be written as $\Delta E_F = -\left({}^T X^{n+1/2} - {}^T X^{n-1/2}\right) P_F^n$, where

$$P_F^n = \bar{P} \tag{17}$$

and $\bar{P}$ takes one of the following values

$$\bar{P} = P^{n-1/2} \tag{18a}$$

$$\bar{P} = P^{n+1/2} \tag{18b}$$

$$\bar{P} \sim \frac{P^{n-1/2} + P^{n+1/2}}{2} \tag{18c}$$

$$\bar{P} \sim \frac{1}{\Delta t} \int_{t^{n-1/2}}^{t^{n+1/2}} P(t) dt \tag{18d}$$

corresponding respectively to a forward-Euler, a backward-Euler, a second-order implicit and a highly-subcycled fluid scheme. For these choices, the parameter δE_F is then given by

$$\delta E_F\,[(17/18a)] = k\left[\frac{h}{2}-\frac{h^3}{48}\right]+d\left[-1+\frac{h^2}{24}\right]+O(h^4) \tag{19a}$$

$$\delta E_F\,[(17/18b)] = k\left[-\frac{h}{2}-\frac{h^3}{16}\right]+d\left[-1+\frac{h^2}{24}\right]+O(h^4) \tag{19b}$$

$$\delta E_F\,[(17/18c)] = k\left[-\frac{h^3}{24}\right]+d\left[-1+\frac{h^2}{24}\right]+O(h^4) \tag{19c}$$

$$\delta E_F\,[(17/18d)] = k\left[-\frac{h^3}{24}\right]+d\left[-1-\frac{h^2}{24}\right]+O(h^4) \tag{19d}$$

Reciprocally, for each possibility for $\bar{P}$ in (18), the input force P_S^{n+1} for the trapezoidal rule (4) can be chosen according to one of the two following choices

$$P_S^{n+1} = \bar{P} \tag{20}$$

$$P_S^{n+1} = 2\,\bar{P}-P_S^n \tag{21}$$

We now give the corresponding parameter δE_S for all these pressure choices. They read

$$\delta E_S\,[(20/18a)] = k\left[-\frac{3h}{2}+\frac{13h^3}{16}\right]+d\left[1-\frac{31h^2}{24}\right]+O(h^4) \tag{22a}$$

$$\delta E_S\,[(20/18b)] = k\left[-\frac{h}{2}+\frac{5h^3}{48}\right]+d\left[1-\frac{7h^2}{24}\right]+O(h^4) \tag{22b}$$

$$\delta E_S\,[(20/18c)] = k\left[-h+\frac{11h^3}{24}\right]+d\left[1-\frac{19h^2}{24}\right]+O(h^4) \tag{22c}$$

$$\delta E_S\,[(20/18d)] = k\left[-h-\frac{3h^3}{8}\right]+d\left[1-\frac{17h^2}{24}\right]+O(h^4) \tag{22d}$$

$$\delta E_S\,[(21/18a)] = k\left[-h+\frac{5h^3}{24}\right]+d\left[1-\frac{13h^2}{24}\right]+O(h^4) \tag{23a}$$

$$\delta E_S\,[(21/18b)] = d\left[1-\frac{h^2}{24}\right]+O(h^4) \tag{23b}$$

$$\delta E_S\,[(21/18c)] = k\left[-\frac{h}{2}+\frac{5h^3}{48}\right]+d\left[1-\frac{7h^2}{24}\right]+O(h^4) \tag{23c}$$

$$\delta E_S\,[(21/18d)] = k\left[-\frac{h}{2}+\frac{h^3}{16}\right]+d\left[1-\frac{5h^2}{24}\right]+O(h^4) \tag{23d}$$

Amongst all possible combinations, very few give second- or third-order energy accuracy. A second-order energy-accurate algorithm is obtained for the fluid force (17/18a) with one of

the input forces (20/18b), or (21/18c), or (21/18d); another possibility is the joint use of a highly subcycled fluid yielding (17/18d) and the input force (21/18b). In that last case, $\delta E = -d\ h^2/12 + O(h^3)$. The only third-order energy-accurate combination is defined by (17/18c) and (21/18b); it does not allow subcycling at all.

2.6 General comparison of partitioned procedures

The class of synchronous staggered schemes of section 2.1 seem to be very accurate when a second-order prediction for the structural displacements is used ($\alpha_0 = 1$ and $\alpha_1 = 1/2$ in (1)). The fundamental result of the evaluation of these methods is that, for any fluid time scheme, there is a way to achieve high energy accuracy. More precisely, provided the forces exchanged between the fluid and the structure at their interface are exactly opposed (16), the coupling algorithm is at least third-order energy-accurate. This is valid for any time scheme (explicit/implicit, first- or second-order time-accurate, subcycled or not).

On the contrary, the proposed asynchronous staggered method described in section 2.5 can lead to a third-order energy error, but for some particular scheme combination. It is also difficult to find a general principle as above, explaining why some scheme combinations should yield less artificial energy production. However, this asynchronous procedure conserves velocity continuity at the interface, a property that was not taken into account by the evaluation introduced in this paper.

In the sequel, we discuss on two- and three-dimensional results the validity of the above evaluations. We compare, for most energy-accurate coupling schemes, the actual behaviour of coupling algorithms with the behaviour predicted using our evaluation method.

3 Numerical Results

3.1 Supersonic panel flutter (two-dimensional)

We consider the aeroelastic response of a flat panel with infinite aspect ratio in a supersonic airstream [1, 5]. The physical problem is only two-dimensional. The panel (Fig. 1) has one side exposed to an airstream and the other side to still air. The panel considered here has a length $L = 0.5\ m$, a uniform thickness $h = 1.35\ 10^{-3}\ m$, a Young modulus $E = 7.728\ 10^{10}\ N/m^2$, a Poisson ratio $\nu = 0.33$ and a density $\rho_S = 2710\ Kg/m^3$. It is clamped at both ends ($x = 0$ and $x = L$). The pressure of the still air under the panel is P_∞ (fluid pressure at infinity).

The fluid is inviscid. The boundary condition at the fluid/structure interface is a slip condition, while at infinity, the fluid is assumed constant (pressure $P_\infty = 25714\ Pa$, density $\rho_\infty = 0.4\ Kg\ m^{-3}$, user-set Mach number M_∞). The simplified analytical study on the linear instability of the panel [1] is based upon the shallow shell theory and a first-order approximation of the aerodynamic theory where the influence of three-dimensional aerodynamic effects is neglected (this approximation is valid for $M_\infty > 1.6$). When the structural vertical displacement U is small, the fluid pressure forces on the panel can be approximated